Atypical Math Course

- Emphasis on working together in groups, sharing work and thought processes as a class, focus on explanations and on fundamental reasoning
- We don't just learn *how to do* basic mathematics, but we explore the building blocks and *why* mathematics works and is useful

<u>My goals</u>

- Provide focused lectures and handouts designed to explore major concepts
- Get you, the students, interested and working **together** on problems
- Provide you with tools, homework, and exams that evaluate your progress
- Host discussions and guide you through learning

Chapters Covered

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Chapter 1

Reasoning About Quantities

1.1 - Ways of Thinking About Story Problems

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In a research study, Dana, a seventh-grader in a program for gifted students in mathematics, was asked to solve the following problem:

> A carpenter has a board 200 inches long and 12 inches wide. He makes 4 identical shelves and still has a piece of board 36 inches long left over. How long is each shelf?

Dana tried to solve the problem as follows: She added 36 and 4, then scratched it out and wrote 200 × 12—but she thought that was too large, so she scratched it out. She then tried 2400 − 36, which was also too large, and discarded it. Next, she calculated 4 × 36 and subtracted that from 200, getting 56. She then subtracted 12 and got 44.

Question - How do you think Dana solved the problem?

> Dana tried all the mathematical operations on pairs of numbers and tried to choose which answer seemed to be the right size.

A study identified seven main strategies that average to above average 6-8th grade students use to solve word problems...

1. Find the numbers in the problem and just do something to them, usually addition because that is the easiest operation.
1. Guess at the operation to be used, perhaps based on what has been most recently studied.
1. Let the numbers “tell” you what to do. (One student said, “If it’s like 78 and maybe 54, then I’d probably either add or multiply. But if the numbers are 78 and 3, it looks like a division because of the size of the numbers.”)
1. Try all the operations and then choose the most reasonable answer. (This strategy often works for one-step problems but rarely works for two-step problems.)
1. Look for “key” words to decide what operation to use. For example, “all together” means to add.
1. Narrow the choices, based on expected size of the answer.
1. Choose an operation based on understanding the problem. (Often students would make a drawing when they used this strategy).

Question - Which strategy do you think is the most mature strategy?

Question - Which of the strategies do **you** use when solving math problems?

1.1 - Ways of Thinking About Story Problems

The children who used strategy #7 used it because they understood the **relationships among the quantities** in the situation.

Definition: A **quantity** is anything (an object, event, or quality) that can be measured or counted. The value of a quantity is its measure of the number of items that are counted. A value of a quantity involves a number and a unit of measure or number of units.

1.1 - What is a Quantity?

Consider the following questions:

- How long do humans live?
- How long do dogs live?
- How far is it to the trash can in this room?
- How long does it take to drive to Los Angeles?
- How far is it to Los Angeles?
- How long does it take to bake brownies?
- How big is this classroom?
- How big is the screen in this classroom?
- How big is an ant?
- Which is bigger, the Earth or the Sun?
- How far is it around the earth?

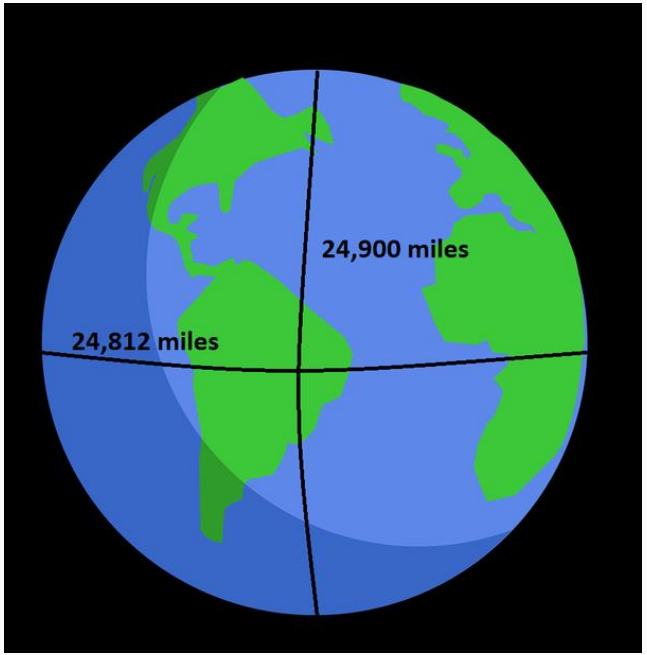
1.1 - What is a Quantity?

~~Consider:~~ From the question:

How far is it around the earth?

What is the "quantity"?

A. 40,000
B. Kilometers
C. 25,000
D. Miles
E. 40,000 km
F. 25,000 miles
G. Distance around the equator



In applied fields quantities typically have specific names. In this case, the distance around the equator is called the meridional circumference.

Quantity, Value, Units

So, associated with the question “How many cookies did Professor Havens eat today?” there is an answer, a quantity, and associated values and units.

- Answer … Kyle ate 37 cookies today.

Question - What is the **quantity**?

Amount of cookies Kyle ate today

Question - What is the **value** of the quantity?

37

Question - What are the associated **units**?

Cookies

Pro Tip - A quantity is never just a number or unit. You can often begin using “amount of” or “number of” or etc. Certain quantities are more specific and start with “distance” or “area” or “time that” or etc. The quantity should also be specific to the situation and not “amount of cookies” but not include the value “37 cookies.”

1.1 - What is a Quantity?

Now let's try to identify the quantity or quantities in the previous questions as well as an appropriate unit of measure.

- How long do humans live?
- How long do dogs live?
- How far is it to the trash can in this room?
- How long does it take to drive to Los Angeles?
- How far is it to Los Angeles?
- How long does it take to bake brownies?
- How big is this classroom?
- How big is the screen in this classroom?
- How big is an ant?
- Which is bigger, the Earth or the Sun?

Note - It can be hard to focus on the **quantity** because we are trained to focus on the **answer**!

1.1 - What is a quantity?

Definition of “Quantify”
express or measure the quantity of

Not all quantities can be “quantified”

Young children often ask “How much do you love me?” but clearly love is not a quantity.

Question: Can you think of other quantities that can’t be quantified?

Discussion - Easy or Difficult to **Quantify**?

Many of the following are easy to quantify. Others are not so straightforward. Of the following, which are easy to quantify and which are not?

- The weight of a newborn baby
- Student achievement
- Blood pressure
- Livability of a city
- Teaching effectiveness
- Human intelligence
- Wealth of a nation

Another Question - How does our society typically quantify the above bullet points?

Group Discussion

Return to your groups. Make a list as you consider the following questions.

- What sorts of events and objects do you think primitive humans felt a need to quantify?
- How do you think primitive societies kept track of the values of those quantities?
- What way(s) might they have quantified wealth?
    - After you have thought of some answers, write them on the board.

1.2 – Quantitative Analysis

- Definition: A **quantity** is anything that can be measured or counted.
  - The **value** of a quantity is its measure of the number that are counted.
  - A value of a quantity involves a number and a **unit** of measure.

- To apply a conceptually sound problem solving strategy, we must try to understand the problem and use our understanding to solve. To fully understand, we must be able to **understand all the quantities** in the problem.

1.2 – Quantitative Analysis

- **Quantitative analysis** is a procedure which we use to aid in solving problems. It starts by naming as many quantities as you can that are involved in the situation. Some essential quantities may not be explicitly stated (they may be implied relationships between other quantities). Also, even if the value of the quantity is not known or given, the quantity is still a part of that situation's quantitative structure.

1. Make a list of the quantities identified in the problem. If the value is not given, indicate that the value is unknown and write down the unit you would use to measure it.
2. Make a drawing that illustrates the problem. The drawing can be unique. It is meant to assist you in solving the problem.
3. Use your drawing to solve the problem.

When working in groups

- Groups of size 3-4 are shown in studies to be most effective modes of learning
- Listen to your neighbors **actively**
- Make sure everyone has the opportunity to provide input
- Be helpful as opposed to critical
- When presenting your groups' work to the class
  - Your problems don't always have to be perfect or finished
  - The goal is to replicate and explain your thought process
  - You become the teacher

Group Activity - Use Quantitative Analysis

- Get in a group with your nearby neighbors to solve the following problem (handout):
  - Two women, Laura and Gloria, each have a brother, Ben and Jerry, respectively. The two women argued about which woman stood taller over her brother. It turned out that Laura won the argument by a 17-cm difference. Laura is 186 cm tall. Ben is 87 cm tall. Gloria is 193 cm tall. How tall is Jerry?
- Present solutions on the board or document camera
  - *Remember* – we are focused on using quantitative analysis in this class, NOT finding the correct answer.
- What is the "key" quantity in this situation?

Having Trouble? Quantitative Analysis Tips

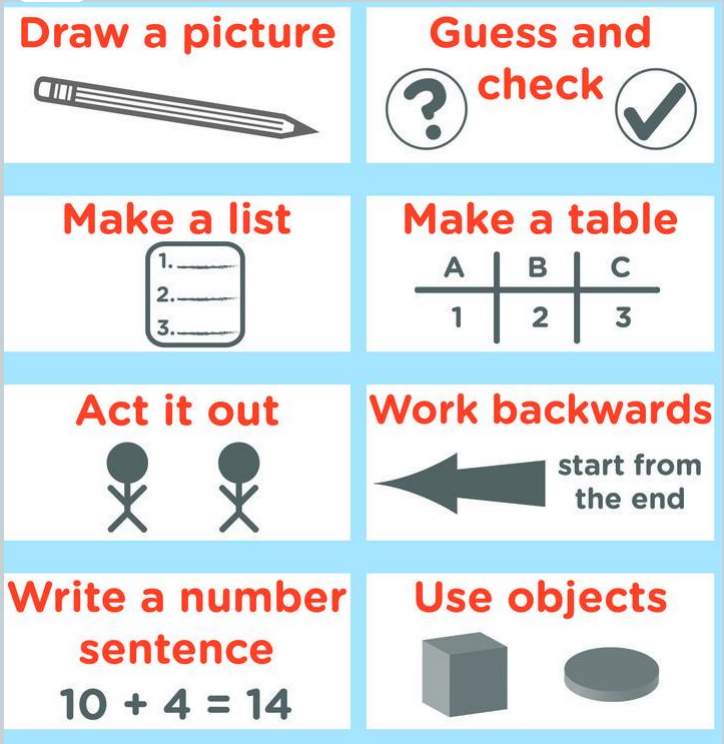
- Can I imagine the situation, as though I am acting it out?
- What relationships will drawing the situation help describe?
- What is the quantity **of interest**? In other words, what quantity, value, and units am I asked to find?
- Which quantities in the problem can help determine the quantity of interest?

Look Ahead – Values of Quantities

- The value of a quantity may involve very large or very small numbers. The value can also involve many types of different numbers, such as whole numbers, fractions, and decimals.
- Large or small?
  - Distance between two stars
  - Diameter of a penny
  - Weight of an aircraft carrier
  - Size of a piece of paper
    - Can you think of three different ways to quantify this?

1.3 – Problem Solving

- If you look for general advice on problem solving, you can find a variety of advice.
  - For example →
- One strategy often overlooked is to try looking at simpler related problem.
  - "Change the numbers"



Coin Problem

- Ethel walks into Best Buy to purchase an xbox for her grandson. The xbox costs $200. She dumps a pile of coins consisting of dimes and nickels on the counter claiming to be exactly $200. You count 2,406 coins in total on the counter.
- Assuming Ethel is giving you the correct amount, how many nickels did she give you? How many dimes?

“Change the Numbers”

- Let’s say we change the number of items we have to be more manageable, then lets think of how we could solve the problem.
- Cost of the item:
  - $200 → $20
- Number of coins:
  - 2,406 coins → 240 coins

- *Group Activity #2* – Try and solve this simplified situation.

Coin Problem

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Preview 1.4 - Issues for Learning

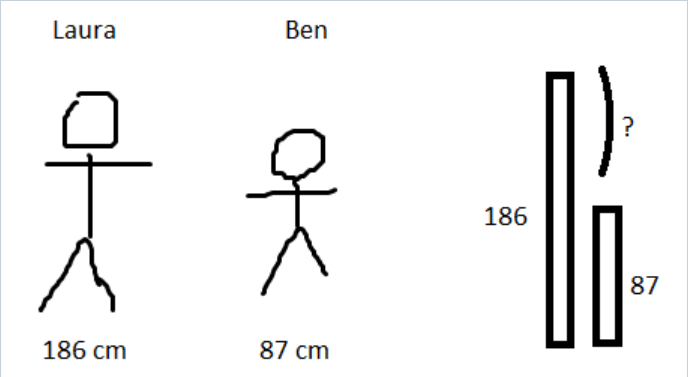
- Ideally we would like children to choose an operation to solve a problem based on a proper understanding of the problem itself.
- As we discussed last class, children (and adults) often use more basic strategies like
  1. Find the numbers in the problem and just do something to them.
  2. Guess at the operation
  3. Let the numbers "tell you" what to do
  4. Try all the operations and use the most reasonable answer.
  5. Make a choice dependent on the expected size of the solution.

1.4 – Issues for Learning: Ways of Illustrating Story Problems

- *Review* – Why Use Quantitative Analysis?
- It is a strategic way to try and understand the problem.
- Too often students are led to believe that the use of diagrams is a form of cheating, or that such a way of solving problems is juvenile.
- In Japan, teachers often say:
  - "If you can draw a picture, you can solve a problem."

Picture versus Diagram

- *Definition* - A **diagram** is a visual representation that displays information in a spatial layout.
  - There is not a huge difference between a picture and a diagram, but a picture is a more artistic representation while a diagram is more focused on the quantitative relationships in the problem.



Example

A carpenter has a board 200 inches long and 12 inches wide. He makes 4 identical shelves and still has a piece of board 36 inches long left over. How long is each shelf?

$$\begin{array}{r} 200 \\ -\ 36 \\ \hline 164 \\ \div\ 4 \\ \hline 41 \end{array}$$

You subtract 200 from 36 and get 164. Now that number is the length of board pieces he has. Then you divide that by 4 (the number of shelves made), and get 41

Strip diagram

In Singapore children are encouraged to use diagrams known as “strip diagrams.”

Sample

▶ The library bought 576 new books. The library loaned some of them. If 198 were left over, how many of the new books did the library lend? ◀

Strip diagram example

Below is one more case of using a “strip diagram”:

▶ Anna and Suzy have $28.90 all together. Anna has $5.48 more than Suzy. How much money does Suzy have? ◀

2 units = $28.90 − $5.48 = $23.42

1 unit = $ _____

Suzy has $ _____

Group Activity #1

- How would you draw it?
  - Roger spent 1/3 of his tax refund on a jet board and then spent ½ of what was left on a mountain bike. He then bought a $400 engagement ring for his girlfriend. If he has $100 remaining, how much was his original refund?