Chapter 5 – Using Numbers in Sensible Ways

- 5.1 – MENTAL MATH
- 5.2 – COMPUTATIONAL ESTIMATION
- 5.3 – ESTIMATING VALUES OF QUANTITIES
- 5.4 – USING SCIENTIFIC NOTATION
- 5.5 – ISSUES FOR LEARNING
- 5.6 – CHECK YOURSELF

Strategies for mental addition/subtraction

- An idea for mental addition
  - https://www.youtube.com/watch?v=cfxsR6sv4jk
- An idea for mental subtraction
  - https://www.youtube.com/watch?v=Ehxnm7kjO9w
  - How is this explanation limited?

- Other strategies?
  - Breaking groups of numbers into chunks and adding those
    - Requires proficiency in adding double digit numbers in your head
    - Carry over must be accounted for
    - Avoid groups requiring carry over
- Let's practice some

Mental multiplication/division

- Ideas for mental multiplication
  - Javier's method (12*6 and 12*12)
  - Book's different methods
    - 12 × 50 can be thought of as 12 × 5 × 10 = 60 × 10 = 600. This required decomposing 50 into 5 × 10, and then using the associative property of multiplication: 12 × (5 × 10) = (12 × 5) × 10 = 60 × 10.
    - 12 × 50 can be thought of as (10 + 2) × 50 = (10 × 50) + (2 × 50) (using the distributive property of multiplication over addition), which is 500 + 100 = 600.
    - 12 × 50 = (6 × 2) × 50 = 6 × (2 × 50) = 6 × 100 = 600 (using the associative property).
- Ideas for mental division
  - https://www.youtube.com/watch?v=_rzq8U78e6M
- Let's practice some

5.2 – Computational Estimation

- Computational estimation
  - Think about computations you do in your regular life
    - How many times do you need an exact answer?
      - Math class ☺
      - On exams/HW, only estimate if I explicitly say to estimate
    - How many times will an approximation suffice?
      - How much should I tip for dinner? How much will the tax be on my shopping?
      - How much money do I need for groceries? Budgeting?
      - How long will it take me to get to work? How far is it?
      - What time do I need to arrive to my interview? To work? To class?

“Mental” Percent Bar

- People who have trouble with percentages often don’t have a way of modeling the problem in their head
- “Draw” a vertical or horizontal bar in your mind, where the bar represents 100%
- You then the large block into equal pieces so that you can reason with a percentage
  - Where is 10%
  - Where is 20%
  - Where is 50%
  - Where is 25%
  - Where is 33.3%
  - Where is 66.6%
  - Where is 90%
  - Where is 110%

Find each of the above percentages of 120 using the bar method.

Activity – Tip your waitresses

- Do tips easily! Mentally! No phone needed!
  - 10% is easy!!
  - What about 5%? That is half of 10%. Easy!!
- Remember: 15% = 10% + 5%
- Using the above ideas, calculate the tip mentally (**15%**) for the following bills:
  a) Example = $18.20 (example…give strats)
  b) $20.00
  c) $38.00
  d) $250.00
  e) $73.80
  f) $3.71
  g) $37.10

➢ Joke of the Day - Where did the one legged waitress work?

Benchmark

3-7

- Benchmark
  - To develop a feel for the size of different numerical amounts, it is helpful to compare them to things you are familiar with. This comparison amount is called a benchmark.
  - Visualization is the key to doing mental math and estimation.
- How long would it take you to walk to the moon?

EXAMPLE 1

▶ It is 250,000 miles from the earth to the moon. How far is that? ◀

If you know that it is approximately 25,000 miles around the earth along the equator, you could think that traveling 10 times around the earth would be comparable in distance to traveling to the moon.

Or,

It is about 2500 miles by air between New York City and Los Angeles, so you can imagine flying between the two cities 100 times.

Or,

you could think of a place that is roughly 10 miles away from where you are and imagine covering that distance 25,000 times!

Estimation and "benchmarks"

- 10% = 1/10 = dividing by ten = shifting place value
  - Can we estimate the following as 10%?
    - 9%
    - 11%
    - 1/3
    - 101/1001
    - 0.100324014018
- 50% = ½ = dividing by two
- 33.3% = 1/3 = dividing by three
  - Can we estimate the following as 33.3%?
    - 33%
    - 34%
    - 1/3
    - 3/4
    - 3/10
    - 0.349201394

Activity – Mental estimation with percents

- Using MENTAL math, *estimate* to approximate the solutions to the following.

5.2/5.3 – Computational Estimation

- *Review –*
  - It is extremely useful to be able to do math in your head, a.k.a. **mental math**
  - Improving mental math capability will improve **number sense**
  - Often **estimating** is adequate for our daily purposes
- In these sections we will examine different types of rounding and when they might be appropriate.

Example – When Estimation is Adequate

- Say you are at the hardware store buying drawer pulls that cost 55¢ each. If you need 36, how much money will you need?
  - Would estimating be appropriate?
  - How could we calculate an estimate?
  - Should we round the numbers up or down?
  - Which number should we round? Both?

Student Solutions

Your students were asked to estimate the value of 36×55.

- *Shawn:* Round to 40 and 60. $40 \times 60 = 2400$.
- *Jack:* First round down: $30 \times 50 = 1500$. Then round up: $40 \times 60 = 2400$. So it's about in the middle, maybe a little past. So I'd say 2000.
- *Maria:* Rounding both up would make it too big, so I'll round 36 to 40 and 55 to 50. $40 \times 50 = 2000$.
- *Jimmy:* A little more than 36×50, which is $36 \times 100 \div 2$ and that's $18 \times 100 = 1800$. It's about 5×36 more, or about 180 more, so I'll say 1980.
- *Deb:* Rounding both up gives $40 \times 60 = 2400$. Since that's too big, I'll say it's about 2200.
- *Sam:* A little more than $6 \times 6 \times 50$, which is $6 \times 300 = 1800$. So I'll say 1900.

Round Up or Down?

- Which type of rounding is better for computational estimation?
  - What about adding?
  - Subtracting?
  - Multiplying?
  - Dividing?
- When estimating how much money you need for rent, should you round up or down? How about the amount you will need for the full year?
- As you spend money, should you round how much you spend up or down?

Mental Estimation

- Try some mental estimates yourself.
  - Think of a story problem that could represent the computation.

- $27 \times \$1.47$

- $\$11{,}609.31 \div 12$

- $\$60 - \$12.47 - \$3.67 - \$20 - \$6.51$

Results from a study on rounding…

- In a research study[1] many middle school students were asked to estimate 789 × 0.52.
  - Some students rounded 0.52 to 1, rounded 789 to 800, and said their answer was 800 × 1 = 800.
  - Is 1 a good substitute for 0.52? Why or why not?
  - Suppose something cost $0.52, and you bought 789 of these objects. Would $800 be a good estimate of the price?
- In the same study the students were asked to estimate 148.52 + 49.341.
  - Some students said, "148 is almost 150 and 49 is almost 50. And 0.52 is almost 0.6, and 0.341 is almost 0.3. And 0.6 + 0.3 is 0.9. So the answer is 200.9."
  - Does this argument make sense to you? Is there a flaw?

Group Activity – Digging Deeper

- What mathematical properties could help when mentally estimating: $\frac{47 \times 53}{5}$
- If 34 × 86 is estimated as 30 × 86, then the estimate is 2580. The exact answer is 2924. The difference between these two numbers is 344. Similarly, if 496 × 86 is estimated as 500 × 86, then the exact answer is 42,656 and the estimate is 43,000. Again, the difference is 344. *Consider*:
  - Why is the difference the same in each estimation?
  - Which estimate is "better?" The first, the second, or are the equally accurate?
- 28 × 77 could be estimated in the follow ways – (a) 30 × 77 (b) 28 × 80 (c) 30 × 80. Which is the best estimate?
- If we were to estimate 53 × 27 with 50 × 30, is the estimate (a) less than (b) greater than (c) equal to the exact value?

5.4 – Using Scientific Notation for Estimating Large/Small Quantities

Numerosity

- "Numerosity" is a term for describing the absolute size of a number
  - 41
  - 41,000
  - 0.00041
  - 41,000,000,000,000,000,000
- Estimating numerosity
  - Jelly beans in a jar
  - People in a football stadium
  - Number of cells in your body
  - Width of a cell in your body

Numerosity and number sense

- "Absolute size"
  - 41 people
  - $10.00
  - 200 pounds
- "Relative size"
  - 3 versus 30
  - ½ versus ¾
  - 67.4 versus 0.674
  - 0.674 versus 0.00674
- Could you ask third graders to bring in their pennies until they reach a million?
  - How about ten-thousand?
  - One thousand?

Dealing with Large/Small Numbers

- 41,000,000,000,000,000,000,000
- 0.000000000000000000041

- Is there an easier way to write/describe large or small numbers?
- YES! It is called "**scientific notation**"
  - 4.1×10^{19}
  - 4.1×10^{-20}

Scientific notation

A number is written in **scientific notation** when it has the form $a \times 10^b$, where a is a number between 1 and 10, and b is an integer.

- You take the number and express it with a quantity that is between 1 and 10 multiplied by the appropriate power of 10.
  - 41,000,000 → $4.1\,000\,000 \times 10^7$
  - 0.0000041 → $0\,000004.1 \times 10^{-6}$
- How do you find the "appropriate power of ten?" →You count the decimal place difference between the original number and how its written in scientific notation.

Scientific notation

- Which of the following numbers, all of which are equivalent to 21,700, are in scientific notation?
  - 21,700
  - $0.000217 \cdot 10^8$
  - $21.7 \cdot 10^3$
  - $0.217 \cdot 10^5$
  - $2.17 \cdot 10^4$

Scientific Notation

PRACTICE CONVERTING

Calculating with scientific notation

- We can use some laws of exponents to make it easier to multiply and divide numbers while in scientific notation.

Two laws of exponents useful in scientific notation are

$$a^m \times a^n = a^{m+n} \quad \text{and} \quad a^m \div a^n = a^{m-n}$$

- ACTIVITY!

Add/Subtract/Multiply/Divide in Scientific Notation

- $(3.2 \cdot 10^9)(1.5 \cdot 10^7)$
- Estimate - $6{,}303{,}000 \times 468{,}000$
- $\frac{4.5\times10^{11}}{9\times10^5}$
- Estimate - $7{,}230{,}000 \div 36{,}209$
  - $1.8 \times 10^8 + 3.2 \times 10^6$?????