Chapter 6

MEANING FOR FRACTIONS

Opening Discussion

- Compare the quantities
  - 2/5 of the class
  - 12/30 of the class
  - 40% of the class
  - 0.4 of the class
- Compare the quantities
  - 5/4 pounds of cheese
  - 1.25 pounds of cheese
  - 1 ¼ pounds of cheese
  - 125% of a pound of cheese

Fraction vs Fractional Expression

- In general, we call all of the following **fractional expressions**

In general, the term “fractional expression” includes all of the following:

$$\frac{\sqrt{3}}{4} \qquad \frac{3+\sqrt[3]{7}}{5} \qquad \frac{9}{10} \qquad \frac{2}{3} \qquad \frac{1.382}{0.94} \qquad \frac{12\frac{3}{4}}{1\frac{5}{6}} \qquad \frac{2.3\times 10^{3}}{1.7\times 10^{-2}} \qquad \frac{a^{2}}{b}$$

- For this class however, the term **fraction** will usually be limited to the more prominent form seen in elementary school

$$\frac{a}{b}$$ where a is any whole number and b is a nonzero whole number

6.1 - Part-whole characterization

- Chapter 6 focuses on the "**part-whole**" characterization of fractions.

The **part–whole** meaning for the fraction $\frac{a}{b}$ has these three elements:

$$\frac{a}{b}$$

1. The unit, or whole, is clearly in mind. (What equals 1?)
2. The denominator indicates into how many pieces of equal size the unit is cut (or is thought of as being cut).
3. The numerator indicates how many such pieces are being considered.

- Fractions are sometimes also called **rational** numbers

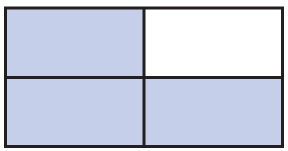
Discrete vs. Continuous

- Quantities which are separate and countable objects are called **discrete**. (Can be counted)
- Quantities which are measured by length, area, mass, are called **continuous**. Continuous quantities cannot be counted. (Can be measured)
- How do these views pertain to fractions?
  - *Discrete* - Typically the whole is the group of countable objects that are considered the unit or team.
    - *Ex*: The soccer team of 24 kids has 16 boys. What fraction of the team do boys make?
  - *Continuous* – Typically the whole is thought of as whatever measurement was made and it can be cut up in any number of equal-sized pieces.
    - *Ex*: A pizza is uncut. How many different ways could be slice it?

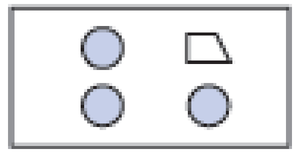
ACTIVITY 1 What Does $\frac{3}{4}$ Mean?

Explain how each of the following diagrams could be used to illustrate the part-whole meaning for $\frac{3}{4}$, making clear what the unit, or whole, is in each case. In which diagrams does $\frac{3}{4}$ refer to a part of a continuous length or region? In which diagrams does it refer to a discrete unit?

a.



b.



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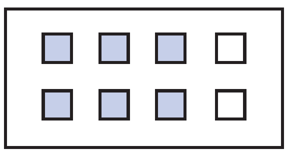


d.

(a) Part = 3 rectangles, Whole = 4 rectangles, Continuous

(b) Part = 3 objects, Whole = 4 objects, Discrete

Notice that this isn't a good way to represent part-whole since the pieces aren't equal

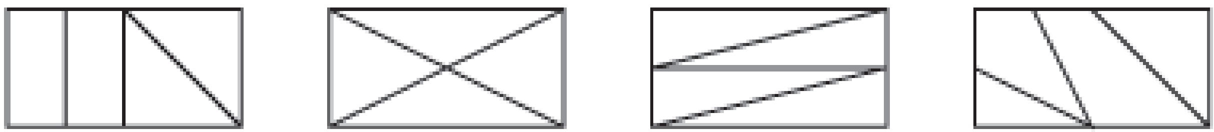
(c) Part = 6 objects, Whole = 8 objects, Discrete

(d) Part = 3 line bits, Whole = 4 line bits, continuous

Same Size?

DISCUSSION 2 When Are the Parts Equal?

Explain how you know that the pieces are or are not the same size.



Careful use of drawings, making certain the pieces have the same area, can help develop an understanding of fractions.

- What about with a discrete quantity?
- What about with a continuous quantity?

Fractions and division

- Is $\frac{7}{8}$ the same as $7 \div 8$?
  - Part-whole model of fractions
    - 7 parts of the whole (8)
  - Sharing equally model of division (RS? MFV?)
    - Sharing 7 brownies with 8 people

- Another comparison: Think about a carton of 12 eggs and dividing by 4 eggs.
  - How would you represent $\frac{12}{4}$ (what is the whole?)
  - How would you represent $12 \div 4$ (think repeated subtraction)
  - How would you represent $12 \div 4$ (think sharing equally)

Perspective?

The fraction $\frac{a}{b}$ can be interpreted to mean **sharing-equally division** (or **partitive division**), denoted by $a \div b$, in which case a wholes would be partitioned into b equal parts, each part being an equal "share."

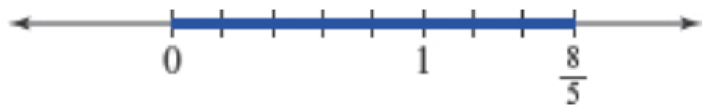
The fraction $\frac{a}{b}$ can also be interpreted to mean **repeated-subtraction division** (**measurement** or **quotitive division**): How many of b are in a? If the fraction can be represented as $\frac{1}{n}$, or $1 \div n$, it is called a **unit fraction.**

Also – Often it is helpful to think of a fraction a just a **single number**.

Fractions on a Number Line

EXAMPLE 1

Model $\frac{8}{5}$ on the number line using this conception. Think $\frac{8}{5}$ means 8 one-fifths, where 5 one-fifths equals one whole.



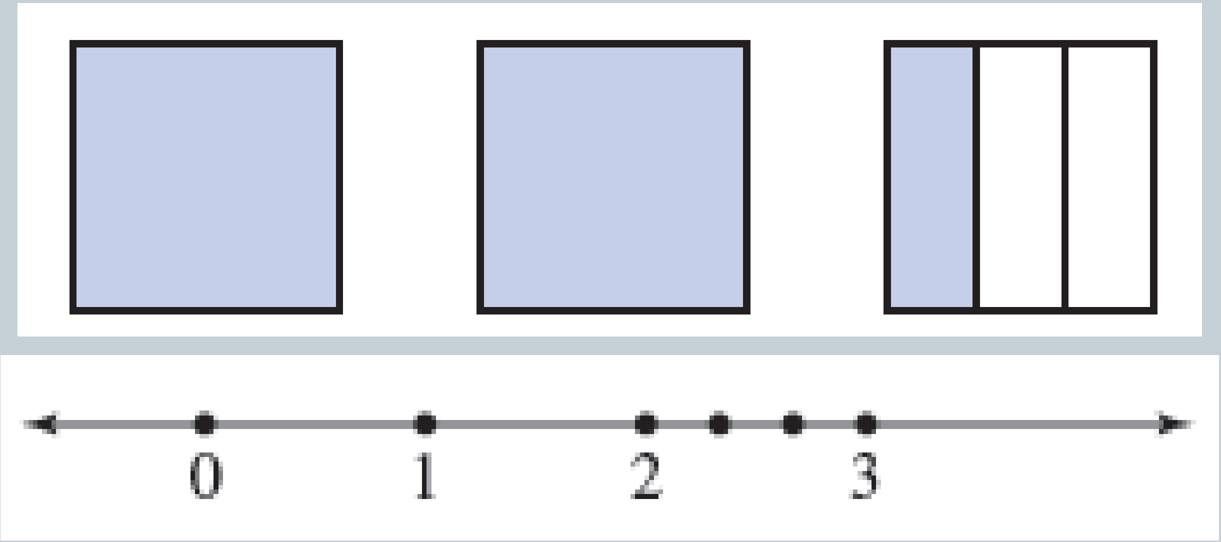
Activity: Draw the following fractions on separate number lines.

$\frac{4}{9}$ $\qquad$ $\frac{11}{10}$

$\frac{42}{9}$ $\qquad$ $-\frac{7}{3}$

Improper Fractions and Mixed Numbers?

- *Video Time –*
  - Watch Rachel “derive” a way to convert.
- Can we make sense of the rule which says that $2\frac{1}{3}$ is equal to $\frac{7}{3}$ using the following diagrams?



Procedural Knowledge vs. Conceptual Knowledge

- Convert $6\frac{3}{8}$ into an improper fraction.
- Convert $\frac{83}{6}$ into a mixed number.
  - Draw a diagram for each conversion and explain why the "conversion algorithm" works.

6.2 – Equivalent Fractions

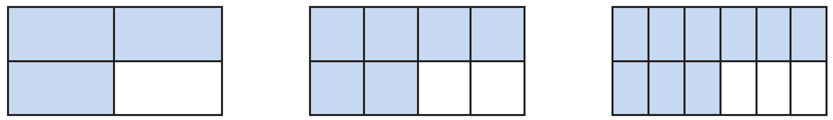
- $\frac{2}{3}$ and $\frac{100}{150}$ look very different to children
- *Video Time –*
  - Let's watch ally compare some fractions and her teacher discuss the issues in the classroom.

Equivalent Fractions

- $\frac{2}{3}$ and $\frac{100}{150}$ look very different to children

As a general principle, $\frac{a}{b} = \frac{a \times n}{b \times n}$ as long as n is not 0.

- $\frac{100}{150} = \frac{10\times10}{15\times10} = \frac{2\cdot5\cdot2\cdot5}{3\cdot5\cdot2\cdot5} = \frac{2}{3}$
  - This approach may make sense to us, but it is not a good place to start with children.
- Here is a better way to explain 3/4 = 6/8 =9/12



Principle of equivalent fractions

- Equivalent fractions will always share a common factor

As a general principle, $\frac{a}{b} = \frac{a \times n}{b \times n}$ as long as n is not 0.

- This rule also makes it **very easy** to find more equivalent fractions
- For the fraction 5/7, create an equivalent fraction using the above rule using
  - N=2
  - N=3
  - N=10
  - N=100

Fraction in "Simplest Form"

- Writing a factor in **simplest form** means removing all the common factors. (for example 2/4 = (1*2)/(2*2) → ½)
- We say that a fraction in simplest form is in **lowest terms**
- Are the following fractions in lowest terms?
  - 2/3
  - 6/8
  - 6/9
  - 12/20
  - 985/250
  - 187/253
  - 102/101

LCM and GCD

The **least common multiple (LCM)** of two (or more) whole numbers is the smallest number that is a multiple of the two (or more) numbers.

For example, 12 is the least common multiple of 3 and 4 because if you list the multiples of 3 and the multiples of 4, it is the smallest number they have in common.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, . . .

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, . . .

The **greatest common factor (GCF or gcf)** of two (or more) whole numbers is the largest number that is a factor of the two (or more) numbers. For example, 2, 3, and 6 are all common factors of 12 and 18, and 6 is the *greatest* common factor of 12 and 18. The GCF is sometimes also called the **greatest common divisor (GCD or gcd).**

Converting Fractions to Decimals

- We know we have multiple ways of writing the same number using the relationship between fractions and decimals. For example:
  - 1/10 = 0.1 = 10%
    - So what decimal would 3/10 be?
    - And what fraction would 0.7 be?
  - ¼ = 0.25 = 25%
    - So what decimal would ¾ be?
    - And what fraction would 1.25 be?
  - 1/3 = 0.3333… repeating = $0.\overline{3}$ = 33.3%
    - So what decimal would 2/3 be?
    - And what fraction would 0.111… be?
    - Then what fraction would 0.222…. be?

Converting Decimals to Fractions

- Converting a decimal into a fraction is EASY!
  - By easy – I mean there is always a direct way to convert it.
- For example, if I wanted to write 0.00267 as a fraction, how could I do it?
  - Remember, a "fraction" must involve only whole numbers
  - So we could use the easiest whole number component of 0.00267
    - 267
  - Now what must I divide 267 by to become equivalent to 0.00267? (hint: it's a power of 10)
    - 267 / 10^5 (since I must move the decimal place 5 spots)
- So the answer is 267/100000
  - Is this in simplest form?

The 2's and 5's rule

- Every power of 10 factors as just 2's and 5's
  - Why?
  - $10 = 2 * 5$ which means that
  - $10^2 = 2^2 * 5^2$ and
  - $10^3 = 2^3 * 5^3$ and so forth.....
- $10^n = 2^n * 5^n$
  - Going back to the last question:
- The answer was 267/100000
  - Is this in simplest form?

Try a few more…

- Convert the following decimals into fractions. Reduce.
  - 0.092
  - 0.0055
- Convert the following fractions into decimals.
  - 3/20
  - 47/25
- Convert the following percent to a fraction.
  - 18%

Repeating decimals

- $\frac{1}{9} = 0.\overline{1}$ → $\frac{a}{9} = 0.\overline{a}$
  - One repeating decimal
- $\frac{1}{99} = 0.\overline{01}$ → $\frac{ab}{99} = 0.\overline{ab}$
  - Two repeating decimals
- $\frac{1}{999} = 0.\overline{001}$ → $\frac{abc}{999} = 0.\overline{abc}$
  - Three repeating decimals
- Etc…
- Random tidbit…

Fun Fact

- $\frac{1}{9} = 0.\overline{1} \rightarrow \frac{a}{9} = 0.\overline{a}$

- So, using the above rule, what if a=9 ?
  - $\frac{9}{9} = 0.\overline{9} = 0.99999...$
  - But $\frac{9}{9} = 1$
  - This means .... $0.\overline{9} = 1$

Representing repeating decimals as fractions

- $0.\overline{4}$
- $0.\overline{04}$
- $0.\overline{24}$
- $0.\overline{104}$
- $0.\overline{0004}$
- $0.0\overline{4}$
  - Notice how this differs from the first few
  - How can we do this? Remember the moving decimal point rule…a decimal shift is the same as multiplying by $0.1 = \frac{1}{10}$
  - So this would be the same as $0.\overline{4} * 0.1 = 0.0\overline{4}$
- $2.12\overline{67}$ = 2.12 + $0.00\overline{67}$ = …

Activity

- Practice some conversion with repeating decimals

- *Recall*
- $\frac{1}{9} = 0.\overline{1} \rightarrow \frac{a}{9} = 0.\overline{a}$
  - One repeating decimal
- $\frac{1}{99} = 0.\overline{01} \rightarrow \frac{ab}{99} = 0.\overline{ab}$
  - Two repeating decimals
- $\frac{1}{999} = 0.\overline{001} \rightarrow \frac{abc}{999} = 0.\overline{abc}$
  - Three repeating decimals
- Etc…

Estimating using fractions

- Using our knowledge of fractions and decimals, we can easily do estimation of the values of fractions
  - 27/206
    - This is about 25/200
    - In simplest form, 25/200 = 1/8
    - 1/8 is about 0.12 or about 12%
  - 123%
    - This is close to 125%
    - That is exactly 1.25
    - That is 1+0.25
    - That is 1 + ¼
    - Which is 5/4