Chapter 7

COMPUTING WITH FRACTIONS

(UNDERSTANDING THE OPERATIONS)

Review

- Last class we talked about
  - Converting between fraction representation and decimal/percent
  - **Terminating** decimals and **repeating** decimals
  - Terminating decimals and “10”s
  - Repeating decimals and “9”s
  - Different types of numbers
    - Rational
    - Irrational
    - (Real? Imaginary? Complex?)

Adding and Subtracting Fractions

Think About . . .

Which of these two calculations is easier? Why?

$$\frac{79}{144} + \frac{35}{144} - \frac{13}{144} \quad \text{or} \quad \frac{3}{8} + \frac{1}{6} + \frac{2}{15}$$

- What does the term common denominator mean?
- Why is it easy to do addition and subtraction with a common denominator?

- The first looks more difficult since the fractions are "larger" (the numbers built into the fractions are bigger), but since you do not have to worry about the denominator at all it makes the addition and subtraction easy.

Adding and Subtracting Fractions

- How do we find a common denominator?
- Most "straightforward" way:
  - Take the product of each of the given denominators
  - Take this product as your new denominator,
  - Each numerator will be the original numerator times the denominators in the **other** fractions

$$\frac{3}{8} + \frac{1}{6} + \frac{2}{15}$$

Algorithm for Adding/Subtracting Fractions

DISCUSSION 1 Algorithms for Adding and Subtracting Fractions

Algorithms for adding and subtracting fractions are sometimes expressed symbolically as follows: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, and $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$. Do they make sense? Apply the first rule to $\frac{3}{8} + \frac{1}{4}$. Is there an easier way to add these fractions than using the rule?

Least Common Multiple / Denominator

- An effective way to find a common denominator is to use the "L.C.M." of the denominators.

A number is the **least common multiple (LCM** or **lcm)** of a set of numbers if it is a common multiple of each of the numbers (i.e., each number in the set is a factor) and it is the smallest number greater than zero with this property. If the set of numbers are all denominators of fractions, then the LCM is often called the **least common denominator (LCD** or **lcd)**.

- The LCM or LCD is effective because it is the simplest shared multiple and thus requires the <u>least work</u>
  - Repeat the previous example by using the LCD instead.
  - *Question:* What is the easiest way to identify the LCD?

Handout

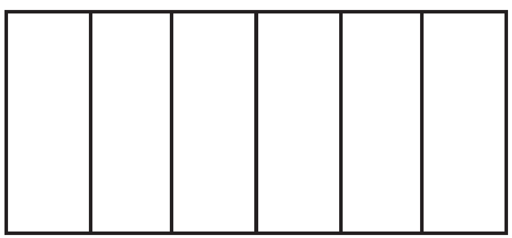
- Let’s explore the operations on fractions by working through the handout for 7.1.

- *Video* – Let’s watch Felisha add fractions and explain.

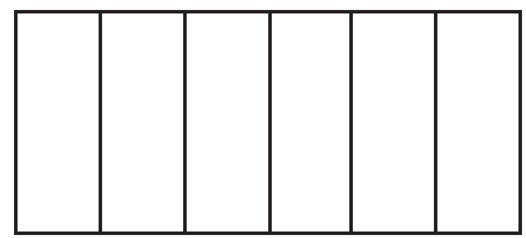
Visually Adding/Subtracting Fractions with a Bar Diagram

ACTIVITY 1 Can You Picture This?

1. Describe how the following drawing can be used to show $\frac{1}{2} + \frac{1}{3}$:



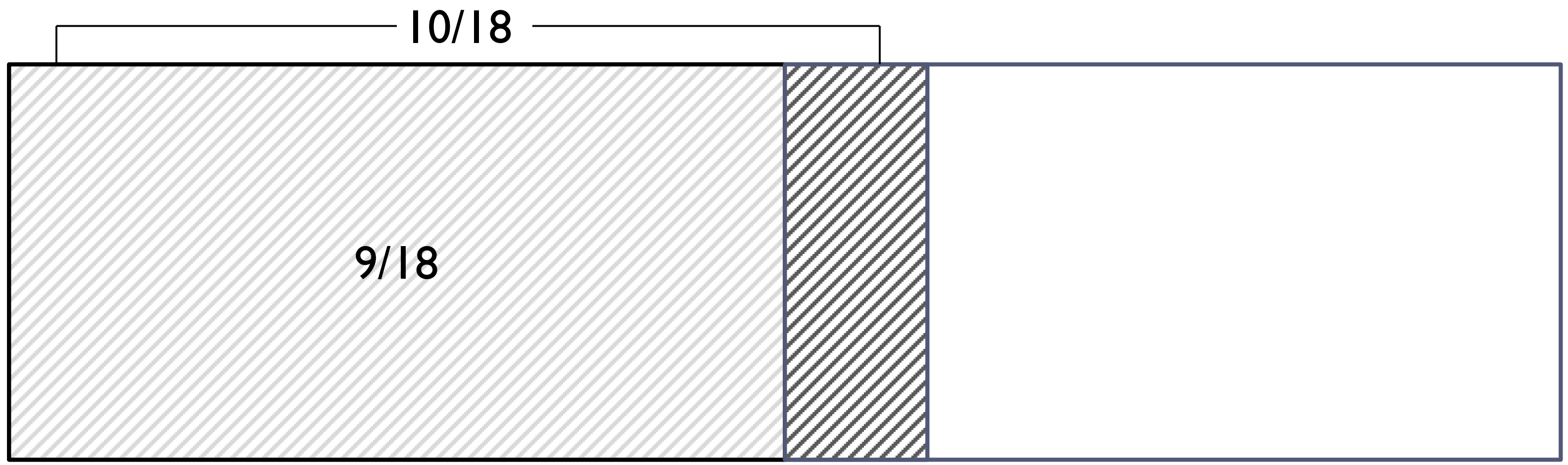
2. Describe how the following drawing can be used to show $\frac{1}{2} - \frac{1}{3}$:



Why did breaking it into sixths make this representation easy?

How many pieces?

- How many pieces should I break the square into if I were trying to show $\frac{5}{9} - \frac{1}{2}$?



$$\frac{5}{9} = \frac{10}{18} \quad \text{and} \quad \frac{1}{2} = \frac{9}{18}$$

7.1 – Adding and Subtracting Fractions

- Algorithm
- Finding the LCD
  - Importance of a common denominator?

ACTIVITY 3 Ancient Problem

Eight loaves of bread are to be shared equally among 10 men. How might this be done? (This problem is from the Rhind papyrus, 1700 B.C.[1]) What part of a loaf does each man get in all?

TAKE-AWAY MESSAGE . . . Addition and subtraction of fractions require understanding that a fraction names a number, a value for a quantity; that fractions can easily be added and subtracted when they refer to a common subunit, which requires that they have a common denominator; and that changing the representation of a fraction to one with the common denominator requires understanding of equivalent fractions.

7.2 – Multiplying by a Fraction

- Does the following rule make sense? Why?

Think About . . .

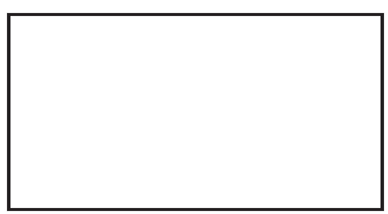
Is there a rationale for the way we multiply fractions:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \cdot c}{b \cdot d}?$$

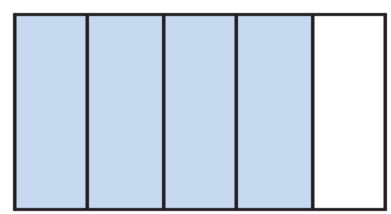
- Let's try this example:

► Juanita had mowed $\frac{4}{5}$ of the lawn, and her brother Jaime had raked $\frac{2}{3}$ of the mowed part. What part of the lawn had been raked? ◄

A drawing might start like this:



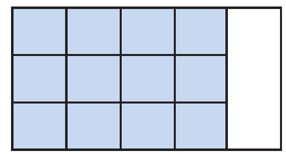
The lawn



$\frac{4}{5}$ of the lawn mowed

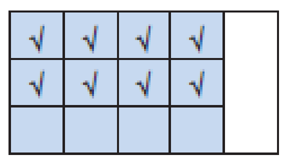
Multiplying by a fraction

Now comes the key step. You could take $\frac{2}{3}$ of the mowed part by first cutting each fifth in the mowed part into three equal parts, as shown below:

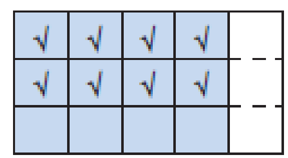


Each fifth cut into three equal parts.

Now it is a matter of taking two of the small pieces in each fifth to get $\frac{2}{3}$ of $\frac{4}{5}$. We will assume a very tidy raker.



The checked part is $\frac{2}{3}$ of $\frac{4}{5}$.



The checked part is $\frac{8}{15}$ of the lawn.

So multiplying by a fraction using the array model shows how you get 8/15. This can be generalized to show how

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \cdot c}{b \cdot d}?$$

Multiplying by a Fraction

- When multiplying fractions using the previous rule, we forget about the **referent unit.** The way to keep it straight is to think about multiplication as an "of" statement.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15} \text{ as } \frac{2}{3} \textit{ of } \frac{4}{5} \textit{ of } 1 \text{ is } \frac{8}{15} \textit{ of } 1.$$

- So the referent unit for each of the following is…
  - R.U. for 2/3 is 4/5
  - R.U. for 4/5 is 1
  - R.U. for 8/15 is 1
- Use this idea to draw the operation on a number line.

More Examples

- Represent the following operations using a number line and the array model.

- $\frac{1}{4} \times \frac{7}{10}$

- $\frac{4}{5} \times \frac{5}{3}$

- $3 \times \frac{2}{5}$

Multiply Makes Bigger?

- Multiply does not always "make bigger."
  - The repeated addition perspective on multiplication can lead to this common misconception
  - Remember that repeated addition fails with fractions (fractional part of a quantity view)
- Multiplying by a fraction always "make smaller?"
  - Yes?
    - When multiplying by a positive "proper" fraction $\frac{a}{b}$ where $b > a$
  - No?
    - When multiplying by an positive "improper" fraction $a > b$.
  - Why? It is always relative to the whole (referent unit).

THINK ABOUT …

1. Some teachers say that multiplication of fractions is easier than addition of fractions. Why might they say that? What are they overlooking?

2. Which of these multiplications, $\frac{1}{3} \times \frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{3}$, means that you begin with $\frac{1}{3}$ of a whole?

7.3 – Dividing by a Fraction

- Let's begin by watching Elliott divide fractions.
  - Video
- Did *you* understand how Elliott did his division?

Dividing by a Fraction

- What is the "easy way" to divide two fractions?

Dividing by a fraction gives the same result as multiplying by the reciprocal of the fraction. Symbolically, when the divisor is a fraction, $n \div \frac{c}{d} = n \times \frac{d}{c}$, or, if n is itself a fraction,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \quad \left(\frac{c}{d} \text{ cannot equal } 0.\right)$$

- The **reciprocal** of a fraction $\frac{a}{b}$ is equal to $\frac{b}{a}$
- We often use the <u>flip and multiply</u> rule to divide fractions.
  - But why?
  - Use the handout as a walk through for the base reasoning.

Chapter 7 – Wrap Up on Operations

- Representing fractions takes practice, but emphasis on the **whole** is critical (as is equal slicing)
- Equivalent fractions are paramount to conceptual understanding operations
  - Adding means "combine"
  - Subtracting means "take away"
    - To do +/- with fractions, it requires a common denominator because different denominators → different piece sizes
  - Multiplication means "scaling up/down"
    - Multiplying by a proper fraction makes smaller (fractional part)
    - Multiplying by an improper fraction makes larger (like whole #s)
  - Division means "how many __ are in __"
    - Dividing by a fraction is opposite to multiply (hence flip&mult)

Comparing the Operations

- *Draw a diagram to model and explain the following.*
  - $\frac{2}{3} + \frac{1}{6}$
  - $\frac{2}{3} - \frac{1}{6}$
  - $\frac{2}{3} \times \frac{1}{6}$
  - $\frac{2}{3} \div \frac{1}{6}$

Issues for Learning: Teaching Fractions

Calculating with fractions is something perceived to be very difficult, in part because it is so poorly understood.

Think About . . .

Would a teacher treat these story problems in the same way pictorially? Numerically?

- For a pizza party, you expect that each of the 12 attendees will eat $\frac{1}{6}$ of a pizza. How many pizzas should you order?
- For a large pizza party, you plan to order 12 pizzas. One-sixth of them should be vegetarian. How many vegetarian pizzas should you order?

- 12 x 1/6 for the first problem, 1/6 x 12 for the second
- You can compare a situation with fractions to one without. See the next slide.
- It is important that you write story problems involving fractions so your students learn to reason with them (and to distinguish add, subtract, multiply, and divide!)

Word Problems Involving Fractions

- Compare a problem involving fractions to a synonymous situation without fractions
  - **Original** = Josh's cheesecake recipe calls for ½ bag of sugar. Josh has 2 ¼ bags of sugar. How many recipes can he make?
    - *What operation is this?*
  - **New** = Josh's cheesecake recipe calls for 2 bags of sugar. Josh has 10 bags of sugar. How many recipes can he make?
- Doesn't the second problem seem easier?
  - "Simplify the problem" – The operations are the same, the only difference are the numbers.

Operations with Decimals

- The four main mathematical operations on decimals are the same with decimals.
  - Though we must review the rules for how to deal with the decimal point
  - Let's also consider – Why do the rules work the way they do?

- Handout