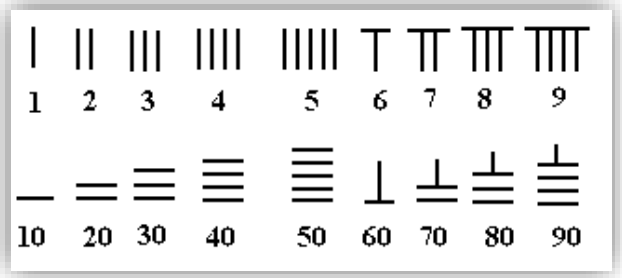
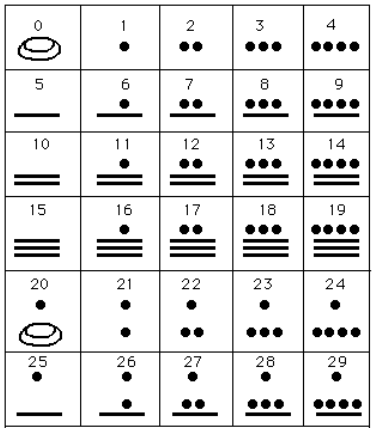
2.1 – Examples of other Numeration Systems

For each of the numeration systems, consider how we'd write the numbers "86" and "206."



1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ϵ	epsilon	50	ν	nu	500	ϕ	phi
6	ς	vau*	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	o	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	φ	koppa*	900	λ	sampi

*vau, koppa, and sampi are obsolete characters



Roman Numeral Table							
1	I	14	XIV	27	XXVII	150	CL
2	II	15	XV	28	XXVIII	200	CC
3	III	16	XVI	29	XXIX	300	CCC
4	IV	17	XVII	30	XXX	400	CD
5	V	18	XVIII	31	XXXI	500	D
6	VI	19	XIX	40	XL	600	DC
7	VII	20	XX	50	L	700	DCC
8	VIII	21	XXI	60	LX	800	DCCC
9	IX	22	XXII	70	LXX	900	CM
10	X	23	XXIII	80	LXXX	1000	M
11	XI	24	XXIV	90	XC	1600	MDC
12	XII	25	XXV	100	C	1700	MDCC
13	XIII	26	XXVI	101	CI	1900	MCM

Group Activity – Place Value

Consider the differences in the following questions.

- What does the numeral “6” in the number “657” represent? The “5?”

- How many tens does the “6” in the number “657” represent?

- How many ten dollars bills does the 6 in $657 represent? The 5?

- How many whole tens are in 657?

- How many tens are in 657?

- How many hundred dollar bills are in $53,908?

- How many whole hundreds are in 53,908?

- What does the “9” in the number “53,908” represent?

- How many hundreds are in 53,908?

- How many pennies can you get for $3.47? $34.70? $347?

- How many tens are in 3,143? How many tenths are in 3,143? How many thousands? Thousandths?

#1 = Andrew's Apple Farm

On Andrew's Apple Farm, very large genetically engineered apples are grown. Andrew collects the apples in little baskets. Eight apples fit in one basket. Andrew has eight baskets. Once all eight baskets are filled, he pours the apples into a bushel. One bushel holds eight baskets worth of apples. Once all eight bushels are filled, Andrew pours the apples from the bushels into a truck. One truck holds eight bushels worth of apples. Andrew has eight trucks.

1. Today Andrew picked enough apples to load one truck, six bushels, and three baskets. How many apples did he pick?

2. If Andrew picks 1705 apples, how many trucks, bushels, baskets, and loose apples would that make?

#2 = Andrew's Apple Books

Andrew keeps careful records of the number of apples that he picks each day. Because he is so used to dealing in trucks, bushels, and baskets, a few years ago Andrew started recording the numbers of apples picked in this form. For example, say Andrew picked enough apples to fill three trucks, one bushel, and seven baskets, and has five more loose apples. He would write:

3	1	7	5
trucks	bushel	baskets	apples

Since he always writes the numbers of trucks, bushels, baskets, and loose apples in order from big to small (left to right), he realized awhile back that he didn't need to write the words out every time. He can just write the digits in order and understand what is meant:

3 1 7 5

The only problem with this notation is that sometimes Andrew shows people his book and they think that the numerals mean what they normally would for us in base ten. The numeral above, for example, looks like it represents three thousand one hundred seventy-five. In order to avoid this confusion, Andrew now puts a little *A* next to any numeral that he writes using his special system:

3175_A

Since he sometimes records other numbers in his books, Andrew knows that if there is not an *A* next to a numeral, it is written in base ten. Sometimes he also indicates base ten to eliminate any ambiguity. For example, he would write:

1661_{ten} or just 1661

This is the same number of apples as three trucks, one bushel, seven baskets, and five loose apples. It's just written differently. Andrew knows that:

$3175_A = 1661_{ten}$

Make sure that you can convert back and forth between these two numeration systems. You should be able to answer the following:

- $2173_A =$ _____$_{ten}$
- $2173_{ten} =$ _____$_A$

Make up your own problems to practice with. Convert in both directions to check your answers.

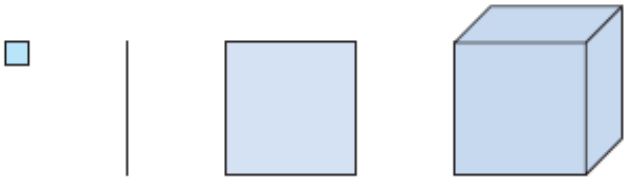
2.3 – Bases Other Than Ten

Different bases are different ways of counting (amount of counting numbers is equal to the base number). They also provide insight into the place value system and how it works in other bases than ten. *Tip*: In a base, there is no digit to express that value. For instance, in our normal system, base 10, there is no symbol for ten. So, in base six for example, there would be no "6."

Example: How do we count in the following bases?

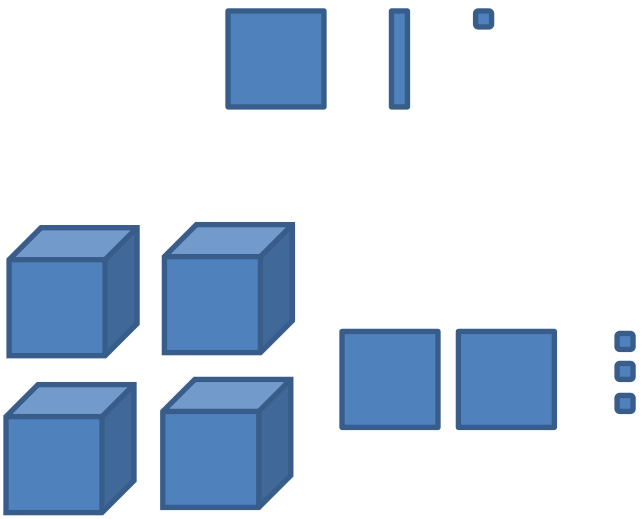
- Base six:
- Base eight:
- Base two:
- *Try it yourself* – Base four:

Base Block Diagrams



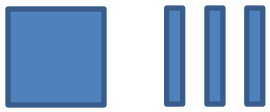
These are very helpful in developing the place value system in a conceptual way. We will use them primarily to try and understand other bases. What does each represent in base ten? Base six? Base eight?

Base Five **Base Ten**



Base Five

Base Ten



Practice Converting from Base Ten into Base Five

1. 18

2. 36

3. 54

4. 127

5. 2016

6. 3600

7. 5616

Practice converting from base 7 into base 10

1. 3_{seven}
2. 12_{seven}
3. 36_{seven}
4. 54_{seven}
5. 66_{seven}
6. 245_{seven}
7. 2013_{seven}
8. 35602_{seven}

There is a more "procedural" way to do this but it isn't of primary focus to us.

Question: Can fractions be represented in other bases? Decimals? Discuss the following.

$$40.13_{\text{five}}$$

Question: Can we "operate" on numbers in other bases? I.E. add, subtract, multiply, divide?

Adding and Subtracting in Different Bases

Side 1…use base block diagrams

1. 31 + 21 (in base 5)

2. 63 + 36 (in base 7)

3. 102 + 221 (in base 3)

4. 327 + 452 (in base 8)

5. 272 - 88 (in base 9)

6. 2004 - 310 (base 6)

Now try the operations using traditional algorithms. What adjustments do we need to make?

1. 341_{five} + 120_{five}

2. 101_{two} + 110_{two}

3. 95_{ten} + 27_{ten}

4. 321_{four} - 123_{four}

5. 201_{three} - 12_{three}

6. 298_{ten} - 29_{ten}