Chapter 5 – Using Numbers in Sensible Ways

5.1 – Mental Math

Try to find the answer to each of the following computations **without writing anything down** or using a calculator.

After you obtain the answer, write down the process you went through to come up with each answer. The focus is on the process, not the answer itself. If you can't do it, discuss potential strategies with your group.

- 152 – 47

- 1000 – 729

- 25 × 24

- 38.6 + 27.2 + 4.8 − 38.6

- 208 ÷ 8

- 6 × 43 + 6 × 7

Discuss as a class the strategies for doing the following problems **mentally**.

- $5197 + 2732$

- $1{,}009{,}302 + 29{,}437{,}219$

- $409 - 313$

- $733 - 409$

- $58{,}367 - 13{,}559$

- $1000 - 782 + 14 + 82 - 215$

Discuss as a class the strategies for doing the following problems **mentally**.

- 12×43

- 107×61

- $1{,}211{,}407 \times 302$

- 99×101 (Is this the same as 100×100? Can we deduce why it is so close?

- $450{,}000 \div 15$

- $37 \div 5$

- $6343 \div 7$

Activity 5.2 – Mental percentages and estimation

Instructions: Without using pen and paper, solve the following problems in your head by **ESTIMATING**. Then, talk about and/or write mathematically how you solved it. Then share with your group.

a) A coat is 30% off. It was originally $150. How much is it now?

b) A jacket cost $45.08 after a 49% discount. About how much was it originally?

c) The waitress brings you your dinner bill and it reads $2,152.15. How much tip should you leave if you want to leave her around 15%?

d) A dress, marked $89.99 before the sale, is 40% off. What is the discounted cost? What percent of the original price did you pay?

e) The saleswoman is charging you $178.69 for all the items you bought today at Express clothing. If they say you saved 33%, how much did your items cost before the savings?

Estimating the Value of Quantities (Mentally)

Example: Say you are at the hardware store buying drawer pulls that cost 55¢ each. If you need 36, how much money will you need? Think about the following.

- Would estimating be appropriate?
- How could we calculate an estimate?
- Should we round the numbers up or down?
- Which number should we round? Both?

Imagine that your students provide you with the following estimates.

- *Shawn:* Round to 40 and 60. 40 × 60 = 2400.

- *Jack:* First round down: 30 × 50 = 1500. Then round up: 40 × 60 = 2400. So it's about in the middle, maybe a little past. So I'd say 2000.

- *Maria:* Rounding both up would make it too big, so I'll round 36 to 40 and 55 to 50. 40 × 50 = 2000.

- *Jimmy:* A little more than 36 × 50, which is 36 × 100 ÷ 2 and that's 18 × 100 = 1800. It's about 5 × 36 more, or about 180 more, so I'll say 1980.

- *Deb:* Rounding both up gives 40 × 60 = 2400. Since that's too big, I'll say it's about 2200.

- *Sam:* A little more than 6 × 6 × 50, which is 6 × 300 = 1800. So I'll say 1900.

Which estimates are appropriate for the problem? Which estimates are poor? Why? Which are easiest?

Try some mental estimates yourself. Think of a story problem that could represent the computation.

$27 \times \$1.47$ $\$11{,}609.31 \div 12$ $\$60 - \$12.47 - \$3.67 - \$20 - \$6.51$

Group Activities - Digging a little bit deeper

1. What mathematical properties could help when mentally estimating: $\frac{47\times53}{5}$

2. If 34 × 86 is estimated as 30 × 86, then the estimate is 2580. The exact answer is 2924. The difference between these two numbers is 344. Similarly, if 496 × 86 is estimated as 500 × 86, then the exact answer is 42,656 and the estimate is 43,000. Again, the difference is 344. *Consider*:

    a. Why is the difference the same in each estimation?

    b. Which estimate is "better?" The first, the second, or are the equally accurate?

3. 28×77 could be estimated in the follow ways – (a) 30×77 (b) 28×80 (c) 30×80. Which is the best estimate?

4. If we were to estimate 53×27 with 50×30, is the estimate (a) less than (b) greater than (c) equal to the exact value?

Handout 5.4 – Scientific Notation

A number is written in **scientific notation** when it has the form $a \times 10^b$, where a is a number between 1 and 10, and b is an integer.

Example: Convert the following numbers from decimal notation (normal) into scientific notation.

- 21,700
- 301,040,000
- 0.000082
- 0.01809

Why does this make sense?

Two laws of exponents useful in scientific notation are

$$a^m \times a^n = a^{m+n} \quad \text{and} \quad a^m \div a^n = a^{m-n}$$

Example: Using the law of exponents and rearranging, solve the following problems given in terms of scientific notation. Express your final answer with scientific notation.

- $(3.2 \cdot 10^9)(1.5 \cdot 10^7)$
- $\frac{4.5 \times 10^{11}}{9 \times 10^5}$
- Estimate - $6{,}303{,}000 \times 468{,}000$
- Estimate - $7{,}230{,}000 \div 36{,}209$