Review From Calculus I – Integration and Differentiation

Today we seek to review the most imperative ideas on integration from Calculus I, while discussing terminology, the difference between an antiderivative, integral, and the Fundamental Theorem of Calculus.

THEOREM 1

The **antiderivative** of $f(x)$ is the set of functions $F(x) + C$ such that

$$\frac{d}{dx}[F(x) + C] = f(x).$$

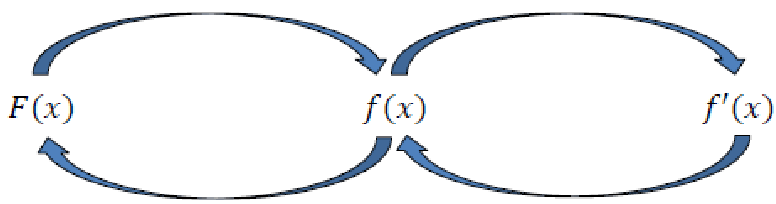
The constant C is called the **constant of integration**.

(x), the antiderivative of $f(x)$, is also commonly written using **integral symbol**.

<u>Indefinite Integral</u>

$$\int f(x)\,dx = F(x) + C$$

<u>Antiderivative of f</u> <u>Original f</u> <u>Derivative of f</u>



Example: For the following functions, identify the antiderivative and derivative of the given function.

$F(x)$	$f(x)$	$f'(x)$
	$2x + 3$	
	x^2	
	$3x^5 - 4x^3$	
	$\sqrt{x}$	
	e^x	
	6^x	
	π^2	
	$\frac{1}{x}$	
	$\ln x$	
	$\sin x$	

Review of Differentiation Rules (Ch. 3)

For each of the following, think about how we might find its derivative and what rule we need to use.

Derivative	Answer?	Rule?
$\frac{d}{dx}(x^2 \sin x) =$		
$\frac{d}{dx}\left(\frac{x^2+4x}{2x-3}\right) =$		
$\frac{d}{dx}\left[e^{x^2}\right] =$		
$\frac{d}{dx}\left(\sqrt[3]{x^2+4x-9}\right) =$		
$\frac{d}{dx}\left(\frac{\sin x}{x}\right) =$		
$\frac{d}{dx}\left[\sin\left(\frac{1}{x}\right)\right] =$		
$\frac{d}{dx}(\tan x) =$		

Basic Applications of Integration and Differentiation

- Differentiation allows us to calculate the instantaneous **rate of change** of the original function
- Integration allows us to calculate the **accumulation** of the original function

In physics, we can use derivatives and antiderivatives to relate a position function $s(t)$ to a velocity function $v(t)$ to an acceleration function $a(t)$.

Example: The velocity of an object in meters per second is given by $v(t) = 4\sqrt{t} + 2t$ from $0 \leq t \leq 16$ seconds.

1. Find the acceleration function $a(t)$.

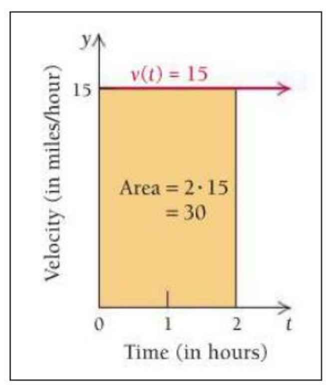
2. Find how rapidly the object is accelerating when $t = 9$.

3. Find the position function $s(t)$ given that $s(9) = 160$.

4. Find the distance the object moved over the first 4 seconds.

Recall - Definite Integral

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



Following these ideas, when an object moves at a certain speed, it accumulates distance based on the speed it travels. It is important to note that the accumulation of a function is the same as the area underneath the graph of the function. Consider a car moving at a constant speed of 15 meters per second. How far will it travel in 2 seconds? A minute? This idea works not only with constant functions but with any continuous integrable function. The described connections are all highlighted and proved by the **Fundamental Theorem of Calculus**.

The Fundamental Theorem of Integral Calculus

If a continuous function f has an antiderivative F over $[a, b]$, then

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx = F(b) - F(a).$$

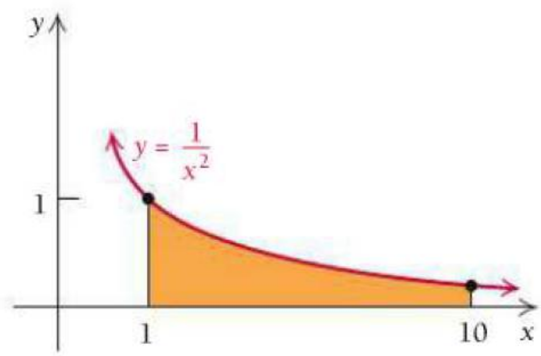
So why is this so fundamental? While formulaically complex, the essence of what it is saying is simple and powerful:

The exact area underneath $f(x)$ between $[a, b]$	The definite integral of $f(x)$ with respect to x	Antiderivative of $f(x)$ → $F(x)$ evaluated at b minus a

This is huge! It tells us that the area underneath the curve can be calculated using a definite integral, which in turn can be calculated by evaluating an antiderivative function i.e. Area Function = Antiderivative Function! Or, more simply stated, if we want to calculate the area underneath a curve we can use a definite integral.

Area Function Original Function Slope Function

Example: Find the area underneath $f(x) = \frac{1}{x^2}$ over $[1,10]$.



Then find the area underneath the same curve from $[2,5]$. Then from $[1,2]$ and from $[5,10]$. What does this imply?