

Formula Sheet Provided

(Will adjust integration tables based on provided questions)

Established integration formulas

$$\int \tan \theta \, d\theta = \ln|\sec \theta| + C \qquad \int \sec \theta \, d\theta = \ln|\sec \theta + \tan \theta| + C$$

$$\int \cot \theta \, d\theta = \ln|\sin \theta| + C \qquad \int \csc \theta \, d\theta = \ln|\csc \theta - \cot \theta| + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \qquad \int \sec^3 \theta \, d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln|\sec \theta + \tan \theta| + C$$

Trig Identities

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta) \qquad \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta) \qquad \tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin 2A = 2 \sin A \cos A \qquad \cos 2A = \cos^2 A - \sin^2 A \qquad \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos^2 x + \sin^2 x = 1 \qquad \sec^2 x = 1 + \tan^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

Integrals of the form $\int \sin^m x \cos^n x \, dx$.

$\int \sin^m x \cos^n x \, dx$	Procedure
n odd	<ol style="list-style-type: none"> Split off a factor of <u>$\cos x$</u>. Apply the relevant identity. Make the substitution $u = \underline{\sin x}$
m odd	<ol style="list-style-type: none"> Split off a factor of <u>$\sin x$</u>. Apply the relevant identity. Make the substitution $u = \underline{\cos x}$
m even and n even	Use the relevant identities to reduce the powers on <u>$\sin x$</u> and <u>$\cos x$</u> .

Integrals of the form $\int \tan^m x \sec^n x \, dx$.

$\int \tan^m x \sec^n x \, dx$	Procedure
n even	<ol style="list-style-type: none"> Split off a factor of <u>$\sec^2 x$</u>. Apply the relevant identity. (all in terms of \tan) Make the substitution $u = \underline{\tan x}$
m odd	<ol style="list-style-type: none"> Split off a factor of <u>$\sec x \tan x$</u>. Apply the relevant identity. (all in terms of \sec) Make the substitution $u = \underline{\sec x}$
m even and n odd	Use the relevant identities to reduce to powers of $\sec x$ alone. Then use integration by parts as in Example 8, page 475.

Expression in the integrand	Substitution
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$
$\sqrt{x^2 + a^2}$	$x = a \tan \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$

Here are some practice questions to test yourself on the Chapter 5 material. The actual exam will not be quite this extensive. You may be given the option to skip a question of your choosing.

1. Evaluate the following integrals.

A. $\int_0^{\sqrt{\pi}} 6t \cos(t^2) dt$

B. $\int x(3x + 1)^5 dx$

C. $\int \frac{e^x}{e^x - 4} dx$

2. Evaluate the following integrals.

A. $\int_1^{81} \sqrt{4x} \ln x dx$

B. $\int \frac{2}{3x^2\sqrt{x^2 - 16}} dx$

C. $\int_0^{\frac{\pi}{3}} \sin^3 \theta - \sin^5 \theta d\theta$

D. $\int \frac{-3x^2 + 5x - 12}{x^3 + 4x} dx$

E. $\int \sqrt{3 - 2x - x^2} dx$

F. $\int_0^{\sqrt{8}} \frac{3x^3}{\sqrt{1 + x^2}} dx$

G. $\int_0^4 (x^2 + 1)e^{-x} dx$

3. Find the area bounded by the curves $y = \cos x$ and $y = \cos^2 x$ between $x = \frac{3\pi}{2}$ and $x = 2\pi$.

4. Evaluate the following improper integrals to determine whether they converge or diverge.

A. $\int_2^{\infty} \frac{2 \ln x}{x^2} dx$

B. $\int_0^1 \frac{1}{2x - 1} dx$

C. $\int_5^{\infty} \frac{8}{x^2} dx$

D. $\int_{-5}^5 \frac{8}{x^2} dx$

E. $\int_4^{\infty} \frac{3}{\sqrt{x}} dx$

F. $\int_0^4 \frac{3}{\sqrt{x}} dx$