Practice Exam #1 Key

<u>Careful</u>: I typed this key up relatively quickly. Hopefully there are no errors, but it is a possibility. Also remember that often there is more than one way to solve these problems but the answer should always be the same in the end.

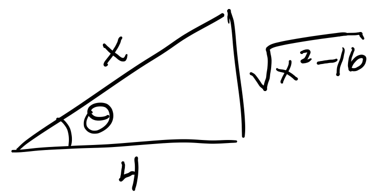
1. (A) u-Sub → $u = t^2, du = 2t\,dt$ → $3\int_0^{\pi} \cos u \; du = [3\sin u]_0^{\pi} = 0 - 0 = 0$

   (B) u-Sub → $u = 3x+1, du = 3\,dx, \frac{du}{3} = dx, x = \frac{u-1}{3}$ →
   $\int \frac{u-1}{3}(u)^5 \frac{du}{3} = \frac{1}{9}\int u^6 - u^5\,du = \frac{1}{9}\left[\frac{u^7}{7} - \frac{u^6}{6}\right] + C = \frac{1}{63}(3x+1)^7 - \frac{1}{54}(3x+1)^6 + C$

   (C) u-Sub → $u = e^x - 4, du = e^x dx$ → $\int \frac{1}{u} du = \ln|u| + C = \ln|e^x - 4| + C$

2. (A) IBP → $u = \ln x\,, dv = \sqrt{x}\,dx$ → $du = \frac{1}{x}dx, v = \frac{2}{3}x^{\frac{3}{2}}$ → $2\int_1^{81}\sqrt{x}\ln x\,dx$
   $= 2\left(\left[\frac{2}{3}x^{\frac{3}{2}}\ln x\right]_1^{81} - \frac{2}{3}\int_1^{81} x^{\frac{3}{2}}\cdot\frac{1}{x}dx\right) = \left[\frac{4}{3}x^{3/2}\ln x\right]_1^{81} - \frac{4}{3}\int_1^{81} x^{1/2}\,dx = \left[\frac{4}{3}x^{3/2}\ln x - \frac{4}{3}\cdot\frac{2}{3}x^{3/2}\right]_1^{81}$
   $= \left[\frac{4}{3}729\ln 81 - \frac{8}{9}729\right] - \left[-\frac{4}{9}\right] = 972\ln 81 - \frac{5824}{9} \approx 3624$

   (B) Trig-Sub → $x = 4\sec\theta\,, dx = 4\sec\theta\tan\theta\,d\theta, \sec\theta = \frac{x}{4}$ →

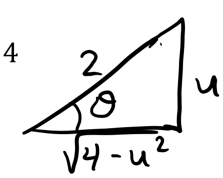


   $$\frac{2}{3}\int \frac{1}{16\sec^2\theta\sqrt{16\sec^2\theta - 16}} 4\sec\theta\tan\theta\,d\theta$$
   $$= \frac{2}{3}\int\frac{4\sec\theta\tan\theta}{16\sec^2\theta\sqrt{16(\tan^2\theta)}}d\theta = \frac{2}{3}\int\frac{4\sec\theta\tan\theta}{16\sec^2\theta\,4\tan\theta}d\theta = \frac{2}{3}\int\frac{1}{16\sec\theta}d\theta$$
   $$= \frac{1}{24}\int\cos\theta\,d\theta = \frac{1}{24}\sin\theta + C = \frac{\sqrt{x^2-16}}{24x} + C$$

   (C) u-Sub → Split off a sine, $u = \cos\theta\,, du = -\sin\theta\,d\theta, -du = \sin\theta\,d\theta, \theta = 0 \to u = 1, \theta = \frac{\pi}{3} \to u = \frac{1}{2}$
   $$\int_0^{\pi/3}\sin\theta\,(\sin^2\theta - \sin^4\theta)d\theta = \int_1^{\frac{1}{2}}((1-u^2) - (1-u^2)^2)(-du)$$
   $$= \int_1^{1/2}(1 - u^2 - 1 + 2u^2 - u^4)\,(-du) = \int_1^{1/2} u^4 - u^2\,du = \left[\frac{u^5}{5} - \frac{u^3}{3}\right]_1^{1/2}$$
   $$= \left[\frac{1}{32\cdot 5} - \frac{1}{8\cdot 3}\right] - \left[\frac{1}{5} - \frac{1}{3}\right] = \left[\frac{1}{160} - \frac{1}{24}\right] - \left[-\frac{2}{15}\right] = \frac{47}{480} \approx 0.979$$

   (D) PFD → $\frac{-3x^2+5x-12}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$ → $-3x^2 + 5x - 12 = Ax^2 + 4A + Bx^2 + Cx$ → $A = -3, C = 5, B = 0$
   $$\int\frac{-3x^2+5x-12}{x(x^2+4)}dx = \int\frac{-3}{x} + \frac{5}{x^2+4}dx = -3\ln|x| + \frac{5}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

   (E) Complete the square, u-Sub, then Trig-Sub. $-x^2 - 2x + 3 = -(x^2 + 2x) + 3 = -(x+1)^2 + 4$
   $u = x + 1, du = dx, u = 2\sin\theta\,, du = 2\cos\theta\,d\theta$



   $$\int\sqrt{3 - 2x - x^2}dx = \int\sqrt{4-(x+1)^2}\,dx = \int\sqrt{4-u^2}\,du = \int\sqrt{4 - 4\sin^2\theta}\,2\cos\theta\,d\theta$$
   $$= \int\sqrt{4\cos^2\theta}\;\,2\cos\theta\,d\theta = 4\int\cos^2\theta\,d\theta = 2\int 1 - \cos 2\theta\,d\theta = 2\left[\theta - \frac{\sin 2\theta}{2}\right] + C = 2\theta - \sin 2\theta + C$$
   $$= 2\theta - 2\sin\theta\cos\theta + C = 2\sin^{-1}\frac{u}{2} - 2\frac{u}{2}\cdot\frac{\sqrt{4-u^2}}{2} = 2\sin^{-1}\left(\frac{x+1}{2}\right) - \frac{x+1}{2}\sqrt{3 - 2x - x^2} + C$$

(F) Trig-Sub OR u-Sub. I chose to do u-Sub. $u = 1 + x^2, du = 2x\,dx, x^2 = u - 1, x\,dx = \frac{du}{2}$

$$3\int_0^{\sqrt{8}} \frac{x^3}{\sqrt{1+x^2}}\,dx = 3\int_0^{\sqrt{8}} \frac{x^2}{\sqrt{1+x^2}}x\,dx = 3\int_1^9 \frac{u-1}{\sqrt{u}}\frac{du}{2} = \frac{3}{2}\int_1^9 u^{1/2} - u^{-1/2}\,du$$
$$= \frac{3}{2}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9 = \left[\sqrt{u}^3 - 3\sqrt{u}\right]_1^9 = [27 - 9] - [1 - 3] = 20$$

(G) IBP → $u = x^2 + 1, dv = e^{-x}dx, du = 2x\,dx, v = -e^{-x}$ → IBP again → $u = x, dv = e^{-x}$

$$[(x^2+1)(-e^{-x})]_0^4 + 2\int_0^4 e^{-x}\cdot x\,dx = [-e^{-x}(x^2+1)]_0^4 + 2\left([-xe^{-x}]_0^4 - \int_0^4 -e^{-x}\,dx\right)$$
$$= [e^{-x}(-x^2 - 1 - 2x - 2)]_0^4 = [e^{-x}(-x^2 - 2x - 3)]_0^4 = [e^{-4}(-27)] - [e^0(-3)] = 3 - \frac{27}{e^4} \approx 2.51$$

3. $\int_{3\pi/2}^{2\pi} \cos x - \cos^2 x\,dx = \int_{3\pi/2}^{2\pi} \cos x - \frac{1}{2}(1 + \cos 2x)\,dx = \int_{3\pi/2}^{2\pi} \cos x - \frac{1}{2}\cos 2x - \frac{1}{2}\,dx$

$$= \left[\sin x - \frac{1}{4}\sin 2x - \frac{1}{2}x\right]_{\frac{3\pi}{2}}^{2\pi} = \left[\sin 2\pi - \frac{1}{4}\sin 4\pi - \pi\right] - \left[\sin\frac{3\pi}{2} - \frac{1}{4}\sin 3\pi - \frac{3\pi}{4}\right]$$
$$= [0 - 0 - \pi] - \left[-1 - 0 - \frac{3\pi}{4}\right] = 1 + \frac{3\pi}{4} - \pi = 1 - \frac{\pi}{4} = \frac{4-\pi}{4} \approx 0.215$$

4. (A) Type 1. Use IBP →$u = \ln x, dv = \frac{1}{x^2}dx$ → $du = \frac{1}{x}dx, v = -\frac{1}{x}$ →

$$2\cdot\lim_{t\to\infty}\int_2^t \frac{\ln x}{x^2}\,dx = 2\cdot\lim_{t\to\infty}\left(\left[-\frac{\ln x}{x}\right]_2^t - \int_2^t -\frac{1}{x}\cdot\frac{1}{x}dx\right) = 2\cdot\lim_{t\to\infty}\left(-\frac{\ln t}{t} + \frac{\ln 2}{2} - \frac{1}{t} + \frac{1}{2}\right)$$
$$= 2\cdot\left(0 + \frac{\ln 2}{2} - 0 + \frac{1}{2}\right) = \ln 2 + 1 \approx 1.69$$ → It converges

(B) Type 2 because this function is discontinuous at $x = \frac{1}{2}$.

$$\int_0^1 \frac{1}{2x-1}\,dx = \int_0^{1/2} \frac{1}{2x-1}\,dx + \int_0^{1/2} \frac{1}{2x-1}\,dx = \lim_{t\to 1/2^-}\int_0^t \frac{1}{2x-1}\,dx + \lim_{s\to 1/2^+}\int_s^1 \frac{1}{2x-1}\,dx$$
$$= \lim_{t\to 1/2^-}\left[\frac{1}{2}\ln|2x-1|\right]_0^t + \lim_{s\to 1/2^+}\left[\frac{1}{2}\ln|2x-1|\right]_s^0 = \left[-\frac{\infty}{2}\right] + \cdots$$ → It diverges

(C) Type 1.

$$8\cdot\lim_{t\to\infty}\int_5^t \frac{1}{x^2}\,dx = 8\cdot\lim_{t\to\infty}\left[-\frac{1}{x}\right]_5^t = 8\cdot\lim_{t\to\infty}\left[-\frac{1}{t} + \frac{1}{5}\right]_5^t = \frac{8}{5}$$

→ It converges.

(D) Type 2 because this function is discontinuous at $x = 0$.

$$\int_{-5}^5 \frac{1}{x^2}\,dx = \int_{-5}^0 \frac{1}{x^2}\,dx + \int_0^5 \frac{1}{x^2}\,dx = \lim_{t\to 0^-}\int_{-5}^t \frac{1}{x^2}\,dx + \lim_{s\to 0^+}\int_s^5 \frac{1}{x^2}\,dx = \lim_{t\to 0^-}\left[-\frac{1}{x}\right]_{-5}^t + \lim_{s\to 0^+}\left[-\frac{1}{x}\right]_s^5$$
$$= \left[\infty + \frac{1}{5}\right] + \cdots$$ → It diverges.

(E) Type 1. It diverges (show work similar to above)

(F) Type 2. It converges to 12 (show work similar to above).