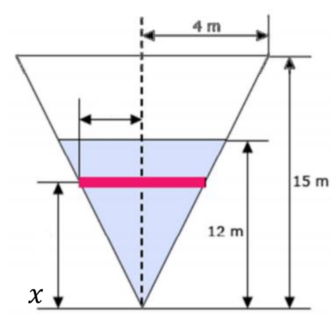
Practice Exam #2

Our second exam will cover all of Chapter 6 and Appendix H. On this exam I will be asking you to explain/sketch.

1. Find the length of the curve $y = \ln(\cos x)$ for $0 \le x \le \frac{\pi}{3}$.
2. Find the slope of the tangent line for the ellipse given by the parametric equation $x = 5\cos t, y = 3\sin t$ when $t = \frac{11\pi}{6}$. Then write the rectangular equation of the tangent line at the location.
3. Find the length of the curve $x = t^2, y = t^3$ between the points $(1,1)$ and $(16, 64)$. Give an exact solution and estimate the amount.
4. Let $r = \sin 3\theta$. Find the slope of the tangent line at $\theta = \frac{\pi}{2}$. Then find the area of one petal.
5. Which of the functions has the largest average value on the interval $[0,2]$? How about on $[2,4]$? $[4,6]$? $f(x) = 2^x, g(x) = x^2$
6. Find the average value of the population of a colony of water buffalo starting with 5 thousand buffalo growing exponentially at a rate of 2% over the course of ten years. **Hint**: $P(t) = 5e^{0.02t}$
7. Radioactive substances decay exponentially. Plutonium-240 is a nuclear isotope in the field of nuclear physics. Plutonium-240 undergoes spontaneous fission that can cause problems with nuclear reactors and nuclear weapons. Say we have 60 mg of Plutonium-240 which has a half-life of approximately 6,560 years. Find the average value of plutonium over the next 1,000 years. **Hint**: $P(t) = 60e^{-0.0001t}$
8. Find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis $y = 1 + x^2,\ y = x + 3$ and sketch the solid of revolution.
9. Let $f(x) = 30x^2(1-x)^2$ for $0 \le x \le 1$ and zero elsewhere. Verify that f is a probability density function and find $P\left(X \le \frac{1}{3}\right)$.
10. Use the cylindrical shells method to calculate the volume of $f(x) = \sin(x^2)$ from $x = 0$ to $x = \sqrt{\pi}$ rotated about the y-axis. Sketch the solid of revolution as well as a typical shell.
11. Find the length of the spiral $r = \theta$ from $\theta \in [0,2\pi]$. Find the area within the spiral on the same interval.
12. Find the volume of the solid obtained by rotating the area underneath $y = \frac{-6}{x-4}$ on the interval $x \in [0,3]$ about the x-axis. Sketch the solid of revolution.
13. Find the volume of the solid obtained by rotating the region bounded by $y = e^{x-1}, y = 0, x = 0, x = 3$ about the x-axis. Sketch. Then find the volume of the same region rotated about the y-axis. Sketch.



14. A water tank is in the shape of an inverted cone with height 15 meters, base radius of 4 meters, and filled to a depth of 12 meters. Let's assume that the force required to raise the volume of water at a point x is given by
$$F(x) = 700\pi x^2(15 - x)$$
Where F is the force in Newtons and x is the distance from the base. Determine the amount of work needed to pump all the water to the top of the tank. Since water exists only over $[0,12]$, integrate that region.

Provided Formulas

$$\int \tan\theta \; d\theta = \ln|\sec\theta| + C$$

$$\int \sec\theta \; d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \cot\theta \; d\theta = \ln|\sin\theta| + C$$

$$\int \csc\theta \, d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\int \sec^3\theta \, d\theta = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Trig Identities

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \qquad \tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A \qquad \tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos^2 x + \sin^2 x = 1 \qquad \sec^2 x = 1 + \tan^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

	Rectangular	**Parametric**	**Polar**
Slope of Tangent Line	$\frac{dy}{dx}$	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$
Area	$A = \int_a^b y \, dx$	$A = \int_\alpha^\beta y \frac{dx}{dt} \, dt$	$A = \int_a^b \frac{1}{2} r^2 d\theta$
Arc Length	$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$	$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$	$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$

Volume of a solid of revolution	
Circular Disks Method	*Cylindrical Shells Method*
$V = \int_a^b \pi[f(x)]^2 \, dx$	$V = \int_a^b 2\pi x f(x) \, dx$

Average Value
$f_{ave} = \frac{1}{b-a}\int_a^b f(x) \, dx$