Practice Exam #2 Key

As usual, I typed this up relatively quickly, so beware of typos or small errors.

1. $\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x \Rightarrow L = \int_0^{\frac{\pi}{3}} \sqrt{1+\tan^2 x}\, dx = \int_0^{\frac{\pi}{3}} \sec x\, dx = [-\ln|\sec x + \tan x|]_0^{\frac{\pi}{3}} = \ln(2+\sqrt{3})$

2. $\frac{dy}{dx} = \frac{3\cos t}{-5\sin t} \Rightarrow \left[\frac{dy}{dx}\right]_{\frac{11\pi}{6}} = -\frac{3}{5}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = \frac{3\sqrt{3}}{5}, \quad x_1 = 5\cos\left(\frac{11\pi}{6}\right) = \frac{5\sqrt{3}}{2}, \quad y_1 = 3\left(-\frac{1}{2}\right) = -\frac{3}{2}$

   $y - y_1 = m(x - x_1) \Rightarrow \quad y = \frac{3\sqrt{3}}{5}\left(x - \frac{5\sqrt{3}}{2}\right) + \frac{3}{2} \Rightarrow \quad y = \frac{3\sqrt{3}}{5}x - 3$

3. $L = \int_1^4 \sqrt{(2t)^2 + (3t^2)^2}\, dt = \int_1^4 \sqrt{4t^2 + 9t^4}\, dt = \int_1^4 \sqrt{(4t^2)\left(1 + \frac{9}{4}t^2\right)}\, dt = 2\int_1^4 t\sqrt{\frac{9}{4}t^2 + 1}\, dt$

   $u = \frac{9}{4}t^2 + 1, du = \frac{9}{2}t\, dt, t \cdot dt = \frac{2}{9}du \Rightarrow \qquad L = \frac{4}{9}\int_{\frac{13}{4}}^{37} \sqrt{u}\; du = \frac{4}{9}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{\frac{13}{4}}^{37} = \frac{8}{27}\left(\left[37^{\frac{3}{2}}\right] - \left[\frac{13^{\frac{3}{2}}}{8}\right]\right)$

   $L = \frac{296\sqrt{37} - 13\sqrt{13}}{27} \approx 64.95$

4. $\frac{dr}{d\theta} = 3\cos 3\theta \Rightarrow \frac{dy}{dx} = \frac{3\cos 3\theta \sin\theta + \sin 3\theta \cos\theta}{3\cos 3\theta \cos\theta - \sin 3\theta \sin\theta}, \qquad \left[\frac{dy}{dx}\right]_{\pi/2} = \frac{3(0)(1) + (-1)(0)}{3(0)(0) - (-1)(1)} = \frac{0}{1} = 0$

   $A = \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2 3\theta\; d\theta = \frac{1}{2}\int_0^{\frac{\pi}{3}} \frac{1}{2}(1 - \cos 6\theta)\; d\theta = \frac{1}{4}\left[\theta - \frac{\sin 6\theta}{6}\right]_0^{\frac{\pi}{3}} = \frac{1}{4}\left(\left[\frac{\pi}{3} - 0\right] - [0 - 0]\right) = \frac{\pi}{12}$

5. $x \in [0,2] \Rightarrow \quad f_{avg} = \frac{1}{2}\int_0^2 2^x\, dx = \left[\frac{2^x}{2\ln 2}\right]_0^2 = \frac{4-1}{2\ln 2} = \frac{3}{2\ln 2} \approx 2.16 \; > \; g_{avg} = \frac{1}{2}\int_0^2 x^2\, dx = \frac{4}{3} \approx 1.33$

   $x \in [2,4] \Rightarrow f_{avg} = \frac{6}{\ln 2} \approx 8.66 \; < \; g_{avg} = \frac{28}{3} \approx 9.33$

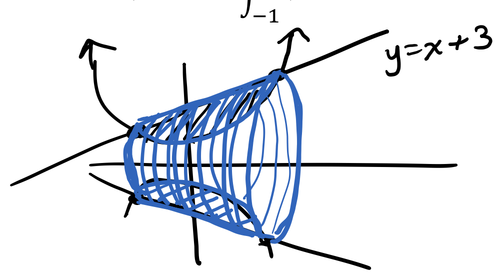
   $x \in [4,6] \Rightarrow f_{avg} = \frac{24}{\ln 2} \approx 34.63 \; > \; g_{avg} = \frac{76}{3} \approx 25.33$

6. $\frac{1}{10}\int_0^{10} 5e^{0.02t}\; dt = \frac{1}{2}\left[\frac{e^{0.02t}}{0.02}\right]_0^{10} = 25[e^{0.2} - 1] \approx 5.54 \Rightarrow$ There is an average of 5,540 water buffalo.

7. $\frac{1}{1000}\int_0^{1000} 60e^{-0.0001t}\, dt \approx 57.1 \Rightarrow$ There is an average of 57.1mg of Plutonium.

8. $V = \pi\int_{-1}^2 (x+3)^2 - (1+x^2)^2\, dx = \pi\int_{-1}^2 (x^2 + 6x + 9 - 1 - 2x^2 - x^4)dx = \pi\int_{-1}^2 (-x^4 - x^2 + 6x + 8)\; dx$

   $= \pi\left[-\frac{x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x\right]_{-1}^2 = \frac{117}{5}\pi \approx 73.51$



9. $\int_{-\infty}^{\infty} f(x)\,dx = 30\int_0^1 x^2 - 2x^3 + x^4\ dx = 30\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1 = 30\left[\frac{1}{30}\right] = 1$

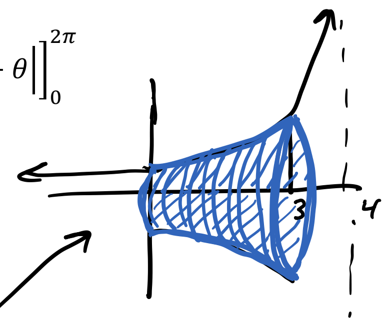
$$P\left(x \le \frac{1}{3}\right) = 30\int_0^{\frac{1}{3}} x^2 - 2x^3 + x^4\ dx = \frac{17}{81} \approx 21\%$$

10. $\int_0^{\sqrt{\pi}} 2\pi x \cdot \sin(x^2)\ dx\,,\quad u = x^2, du = 2x\,dx \Rightarrow V = \pi\int_0^{\pi} \sin u\,du = \pi[-\cos u]_0^{\pi} = \pi([1] - [-1]) = 2\pi$

11. $L = \int_0^{2\pi} \sqrt{\theta^2 + 1^2}\,d\theta,\quad \theta = \tan\phi\,, d\theta = \sec^2\theta\,d\theta\,,\ L = \int_{\theta=0}^{\theta=2\pi} \sqrt{\tan^2\phi + 1}\sec^2\phi\,d\phi = \int_{\theta=0}^{\theta=2\pi} \sec^3\phi\,d\phi$

$$L = \left[\frac{1}{2}\sec\phi\tan\phi + \ln|\sec\phi + \tan\phi|\right]_{\theta=0}^{\theta=2\pi} = \left[\frac{1}{2}\sqrt{\theta^2+1}\cdot\theta + \frac{1}{2}\ln\left|\sqrt{\theta^2+1} + \theta\right|\right]_0^{2\pi}$$

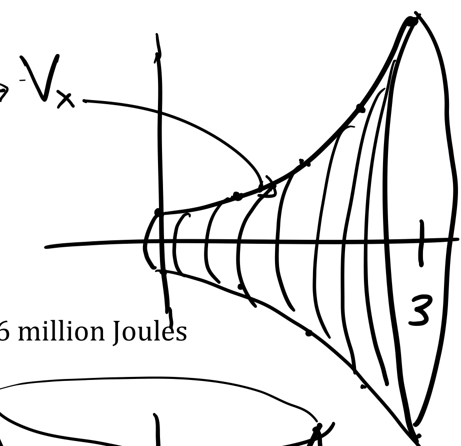
$$= \pi\sqrt{4\pi^2+1} + \frac{1}{2}\ln\left|\sqrt{4\pi^2+1} + 2\pi\right| \approx 21.26$$



12. $V = \pi\int_0^3 \left(\frac{-6}{x-4}\right)^2 dx = 36\pi\int_0^3 \frac{1}{(x-4)^2}dx,\quad u = x - 4, du = dx \Rightarrow$

$$V = 36\pi\int_{-4}^{-1} \frac{1}{u^2}du = 36\pi\left[-\frac{1}{u}\right]_{-4}^{-1} = 36\pi\left([1] - \left[\frac{1}{4}\right]\right) = 36\pi\left(\frac{3}{4}\right) = 27\pi$$

13. $V_x = \pi\int_0^3 (e^{x-1})^2\,dx = \frac{\pi}{e^2}\int_0^3 e^{2x}\,dx = \frac{\pi}{e^2}\left[\frac{e^{2x}}{2}\right]_0^3 = \frac{\pi}{2e^2}(e^6 - 1) \approx 85.6$



$$V_y = 2\pi\int_0^3 xe^{x-1}\,dx = \frac{2\pi}{e}\int_0^3 xe^x\,dx = \frac{2\pi}{e}[xe^x - e^x]_0^3 = \frac{2\pi}{e}(2e^3+1) \approx 95.2$$

14. $W = 700\pi\int_0^{12} 15x^2 - x^3\ dx = 700\pi\left[5x^3 - \frac{x^4}{4}\right]_0^{12} = 2419200\pi \approx 7.6\cdot 10^6 = 7.6$ million Joules