Summary of Convergence Tests for Series

Name	Series	Convergence or Divergence	Comments
Divergence Test (or nth term test)	$\sum a_n$	Diverges is $\lim_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim_{n\to\infty} a_n = 0$
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1} \left(\text{or } \sum_{n=0}^{\infty} ar^n\right)$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$ Diverges if $\lvert r\rvert \geq 1$	* Can be used to calculate sum of series with first term a and common ratio r. * Useful for comparison test with sequences involving r^n.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison test with nth terms involving $\frac{1}{n^p}$ or other rational functions.
Integral Test	$\sum_{n=c}^{\infty} a_n \quad (c > 0)$ where $a_n = f(n)\ \forall\, n$	Converges if $\int_c^{\infty} f(x)dx$ converges Diverges if $\int_c^{\infty} f(x)dx$ diverges	Only works when $f(x)$ obtained from the sequence is continuous, positive, decreasing, and easy to integrate.
Comparison Test	$\sum a_n$ and $\sum b_n$ where $0 \leq a_n \leq b_n$	$\sum b_n$ converges $\Rightarrow \sum a_n$ converges $\sum a_n$ diverges $\Rightarrow \sum b_n$ diverges	Use $\leq$ to compare with a convergent series. Use $\geq$ to compare with a divergent series.
Limit Comparison Test	$\sum a_n$ and $\sum b_n$ $\lim_{n\to\infty} \frac{a_n}{b_n} = c$	$\sum b_n$ converges $\Rightarrow \sum a_n$ converges $\sum b_n$ diverges $\Rightarrow \sum a_n$ diverges	* The terms in both series must be strictly positive. *The obtained c must be positive and finite. * Inconclusive if $c = 0$ or if the limit is infinity or nonexistent.
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$	Converges if (i) $b_{n+1} \leq b_n$ and (ii) $\lim_{n\to\infty} b_n = 0$	* Applicable only to series with alternating terms. * $\lvert a_n\rvert = b_n$
Ratio Test	$\lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert} = L$	Converges absolutely if $L < 1$ Diverges if $L > 1$ or if L is infinite	* Inconclusive when $L = 1$ * Useful when $\sum a_n$ involves factorials or nth powers.
Root Test	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	Converges absolutely if $L < 1$ Diverges if $L > 1$ or if L is infinite	* Inconclusive when $L = 1$ * Useful when $\sum a_n$ involves lots of stuff raised to the n.
Absolute Convergence	$\sum a_n$	$\sum \lvert a_n\rvert$ converges $\Rightarrow \sum a_n$ converges	* Useful for series containing negative and positive terms. * $\sum a_n$ converges absolutely.